CALCULATION OF HEAT TRANSFER IN LAMINAR PIPE FLOW OF LIQUIDS WITH STRUCTURAL VISCOSITY

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ABSTRACT: Expressions have been obtained for the dimensionless heat-transfer criteria of liquids with structural viscosity for the conditions t_W = const and q_W = const in the case of laminar quasi-isothermal flow.

Let us consider the case of quasi-isothermal flow of liquids with structural viscosity, i. e., let us assume that the temperature gradients along the radius of the pipe are such that the heat conductivity, heat capacity, and density of the liquid may be assumed constant over the cross section, while the viscosity depends only on the tangential shear stress τ.

For values of the Peclet number P > 10 in a steady axisymmetric linear laminar flow the energy equation in cylindrical coordinates may be written in the form

$$W\frac{\partial t}{\partial x}=\frac{a}{r}\frac{\partial}{\partial r}\left(r\frac{\partial t}{\partial r}\right), \qquad (1)$$

where W is the flow velocity, t is the flow temperature, and a is the thermal diffusivity.

The general form of the relation between fluidity and shear stress $\varphi(\tau)$ for a liquid with structural viscosity, whose rheological characteristics are independent of time, was proposed in [1]. For liquids with a linear, quadratic, etc., fluidity law the equation for the velocity gradient at $\tau_1 \approx 0$ can be written in the form

$$\frac{dW}{dr}=-\varphi_0\left[1+\sum_{n=1}^{m}\frac{\theta^n}{\varphi_0}\tau^n\right]\tau, \qquad (2)$$

where φ_0 is the zero fluidity (fluidity as $\tau \to 0$) θ is the coefficient of structural stability, and τ is the tangential shear stress. In the region of stabilized flow the shear stress distribution over the pipe cross section has the form

$$\tau=\tau_w\xi \qquad (\xi=r/R), \qquad (3)$$

where τ_w is the tangential shear stress at the wall. Substituting (3) into (2) and integrating for W = 0 when ξ = 1, we get the velocity distribution

$$W=\frac{R\varphi_0\tau_w}{2}\left[1-\xi^2+\sum_{n=1}^{m}\frac{2}{n+2}\frac{\theta^n}{\varphi_0}\tau_w^n(1-\xi^{n+2})\right]. \qquad (4)$$

The expression for the relative velocity has the form

$$\frac{W}{\langle W\rangle}=2\left[1-\xi^2+\sum_{n=1}^{m}\frac{2}{n+2}\frac{\theta^n}{\varphi_0}\tau_w^n(1-\xi^{n+2})\right]\left[1+\sum_{n=1}^{m}\frac{4}{n+4}\frac{\theta^n}{\varphi_0}\tau_w^n\right]^{-1}. \qquad (5)$$

In this case the mean flow rate is equal to

$$\langle W\rangle=2\int_0^1 W\xi d\xi=\frac{R\varphi_0\tau_w}{4}\left(1+\sum_{n=1}^{m}\frac{4}{n+4}\frac{\theta^n}{\varphi_0}\tau_w^n\right).$$

As a rule, media with structural viscosity are characterized by a small coefficient of heat conductivity λ, a considerable specific heat c_p, and a high kinematic viscosity v. Thus, for such media the Prandtl number $\sigma \gg 1$. Therefore we may assume that in these media the thickness of the hydrodynamic boundary layer is much greater than that of the thermal boundary layer, and the process of heat transfer is concentrated in a narrow region near the wall.

We introduce into (5) the new variable y = R − r and, considering only the region near the wall, we neglect the ratio y/R in powers higher than the first (a similar approach to the diffusion problem was made in [2]). Then the velocity distribution near the wall takes the form

$$W=\frac{4\langle W\rangle y}{R}\chi,$$

$$\chi=\left(1+\sum_{n=1}^{m}\frac{\theta^n}{\varphi_0}\tau_w^n\right)\left(1+\sum_{n=1}^{m}\frac{4}{n+4}\frac{\theta^n}{\varphi_0}\tau_w^n\right)^{-1}. \qquad (6)$$

The coefficient χ takes into account the structurally-viscous properties of a medium with linear (n = 1), quadratic (n = 2), etc., fluidity laws.

Since the thermal boundary layer is much smaller than the radius of the pipe, the layer of liquid participating in heat transfer may be assumed plane, and Eq. (1) may be written in the form

$$W\frac{\partial t}{\partial x}=a\frac{\partial^2 t}{\partial y^2}. \qquad (7)$$

Solution for case t_W = const. We introduce the dimensionless quantities

$$\vartheta\equiv\frac{t_w-t}{t_w-t_0}, \quad X\equiv\frac{x}{D}, \quad Y\equiv\frac{y}{D}, \quad P\equiv\frac{D\langle W\rangle}{a}.$$

Here t_0 is the liquid temperature at the edge of the thermal boundary layer, equal to the temperature at the pipe inlet.

Then Eq. (7) becomes

$$8\chi YP\frac{\partial\vartheta}{\partial X}=\frac{\partial^2\vartheta}{\partial Y^2}. \qquad (8)$$

Introducing the new dimensionless variable

$$\eta=Y\left(\frac{X}{\chi P}\right)^{-1/3}, \qquad (9)$$

from Eq. (8) we get the ordinary differential equation

$$\frac{d^2\vartheta}{d\eta^2}+\frac{8}{3}\eta^2\frac{d\vartheta}{d\eta}=0, \qquad (10)$$

the solution of which with boundary conditions

$$\vartheta=0 \quad \text{at} \quad Y=0, \qquad \vartheta=1 \quad \text{at} \quad X=0,$$

has the form

$$\vartheta=\int_0^\eta \exp\left(-\frac{8}{9}\eta^3\right)d\eta\left[\int_0^\infty \text{exy}\left(-\frac{8}{9}\eta^3\right)d\eta\right]^{-1}. \qquad (11)$$

The Nusselt number

$$N_x=\frac{D}{t_w-t_0}\left(\frac{\partial t}{\partial y}\right)_{y=0}. \qquad (12)$$

Substituting into (12) the expression for $\partial t/\partial y$ at y = 0 from (11), we get

$$N_x=1.07\left(\chi P\frac{D}{x}\right)^{1/3}, \qquad (13)$$

and the average value of the Nusselt number of length L

$$\langle N\rangle=1.62\left(\chi P\frac{D}{L}\right)^{1/3}. \qquad (14)$$

For estimating the change in thickness of the thermal boundary layer δ_T we have

$$\delta_T \sim \frac{\lambda (t_w - t_0)}{q(x)} \sim \left(\frac{R^2 x}{\chi P}\right)^{1/3}.$$

For liquids with structural viscosity for which $\chi > 1$ the thermal boundary layer grows more slowly than for ordinary liquids ($\chi = 1$) at the same value of the Peclet number. Therefore the length of the segment of the pipe of which δ_T attains values of the radius will be somewhat greater, i. e., $L_0 \sim \chi RP$.

Since in most practical cases for liquids with structural viscosity the product χP is large, the entire pipe will be covered by the inlet region, in which the assumption that $\delta_T \ll R$ holds.

Solution for case q_w = const. Differentiating Eq. (8) with respect to Y, we have

$$8\chi P \frac{\partial^2 \vartheta}{\partial Y \partial X} = \frac{\partial}{\partial Y}\left(\frac{1}{Y} \frac{\partial^2 \vartheta}{\partial Y^2}\right). \quad (15)$$

Introducing into (15) the heat flux density ratio

$$Q = \frac{\partial \vartheta}{\partial Y}\left(\frac{\partial \vartheta}{\partial Y}\right)^{-1}_{Y=0}, \quad (16)$$

and making the change of variables (9), we get

$$\frac{d^2 Q}{d\eta^2} + \frac{8\eta^3 - 3}{3\eta} \frac{dQ}{d\eta} = 0. \quad (17)$$

The solution of this equation for the boundary conditions Q = 1 at Y = 0, Q = 0 at X = 0 has the form

$$Q = \int\limits_{\eta}^{\infty} \eta \exp\left(-\frac{8}{9}\eta^3\right) d\eta \left[\int\limits_0^{\infty} \eta \exp\left(-\frac{8}{9}\eta^3\right) d\eta\right]^{-1}. \quad (18)$$

Substituting (18) into (16) and integrating, we get the temperature distribution

$$\vartheta = \left(\frac{X}{\chi P}\right)^{1/3} \left\{\eta \left[1 - \frac{\Gamma(2/3, \eta)}{\Gamma(2/3)}\right] + \frac{\exp(-8/9\, \eta^3)}{(8/9)^{1/3}\, \Gamma(2/3)}\right\}, \quad (19)$$

and the local value of the Nusselt number

$$N_x = 1.29\left(\chi P \frac{D}{x}\right)^{1/3}. \quad (20)$$

The average value of the Nusselt number of length L is equal to

$$\langle N \rangle = 1.93\left(\chi P \frac{D}{L}\right)^{1/3}. \quad (21)$$

Thus, the calculations show that the ratio of the heat transfer coefficients of liquids with structural viscosity to the heat transfer coefficients of ordinary Newtonian liquids at identical values of PD/L and for boundary conditions of both the first (t_w = const) and second (q_w = const) kinds will be

$$N / N_0 = \chi^{1/3}.$$

Values of χ calculated for a series of liquids with structural viscosity (1% solution of sodium carboxymethyl cellulose, 1.7% solution of rubber in toluene, M-III bitumen) did not exceed 1.3. It is therefore to be expected that the values of the Nusselt numbers for such media under conditions of quasi-isothermal flow will differ from their values for ordinary liquids by not more than 10–20%.

REFERENCES

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